National Aeronautics and
Space Administration

OH, WHAT A PANE!

An inquiry based activity with a mathematical approach to investigating windows on Earth....and in space.

STUDENT GUIDE

ARES
Astromaterials Research & Exploration Science

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Space Administration

OH, WHAT A PANE!

Written and Developed for the Expedition Earth and Beyond Education Program
Astromaterials Research and Exploration Science (ARES) Directorate
ARES Education Program, NASA Johnson Space Center
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Written by:
Marshalyn Baker, Classroom Teacher
Michele Mailhot, Mathematics Education Specialist
Paige Valderrama Graff, ARES Science Education Specialist

Edited by the following educators, scientists, and staff within the
Astromaterials Research and Exploration Science (ARES) Directorate
at the NASA Johnson Space Center:

Trevor Graff, *Planetary Scientist*
Patricia Huynh, *Administrative Specialist*
Charles Lindgren, *Classroom Teacher*
Timothy McCollum, *Classroom Teacher*
Susan Runco, *Physical Scientist*
William Stefanov, *Senior Geoscientist*
Kim Willis, *Principal Geoscientist*

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OH, WHAT A PANE!

Oh, What Do You See?

How often do you look out the window? What do you see as you look out the window from your classroom? Perhaps you see a city street, a lake, or a cornfield. What if you were sitting in a classroom in a different school, or even a different state? How would your view change? Everyone has a different view as they look out a window, depending on their location.

How would your view change if you were to take a step back from the window? Think about how your view might change if you were riding on the bus or in a car. What about if you were on a plane? Imagine what you might see if you were an astronaut in space observing Earth from a window on the International Space Station! Think about the function or use of windows. Some may be used for viewing in or out, for letting light in, or for decorative purposes. Many windows have multiple uses.

Where you are certainly determines what you see as you look out a window. The type and shape of a window, however, can also change your view. In fact, windows come in many different shapes: rectangles, circles, ellipses (ovals), and octagons. In the "Oh, What A Pane!" activity you will go through a set of exercises, each with a different purpose and use of your mathematical knowledge. You will start by making observations of windows you see every day and will eventually think about the windows astronauts look out of as they orbit the Earth on the International Space Station.

For starters, determine what makes the "perfect" window. Let's investigate what that might be.

1. Observe different buildings that have windows (i.e., a school, a church, a store, a house). Keep a tally of the various shapes and how many of each shape you find. Include where you found each window, its potential function(s) and what percentage each shape is of the total number of windows you observe. An example has been included for you.

Shape of Window	Location(s) and Potential Function(s)	Tallies for the # You See of This Shape	Total # You See of This Shape	Percentage of the Total of All Windows Observed
Example: Rectangle	*Messalonskee Middle School: viewing in/out, letting light in; Oakland Public Library: viewing in/out, letting light in; my house: viewing in/out, letting light in.*	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	40	Can only complete after all window observations have been recorded.
Circle	*Oakland Public Library: viewing in/out, letting light in; my house: viewing in/out, letting light in.*	𝍸	5	Can only complete after all window observations have been recorded.
Other	*Same locations as above and/ or other locations and function(s)*	?	?	Can only complete after all window observations have been recorded.

As you observe different buildings, be sure to record the different shapes, locations, potential functions, and tally the number of windows you see of that shape. You will likely find that different buildings have windows with more than one shape.

Fill in the table below with your observations of windows on different buildings.

Shape of Window	Location(s) and Potential Function(s)	Tallies for the # You See of This Shape	Total # You See of This Shape	Percentage of the Total of All Windows Observed
		TOTAL:		

2. Based on your data, what is the most common shape for a window?

3. Why do you think this is the case?

4. Do you see any patterns in your data between the shape of the windows and what the purpose of the windows might be?

5. How is your data similar to the data of others in your class? How is it different?

One consideration you might have as you determine the “perfect” window could be the size, shape, or area of the window. You will compute the area of windows with the following shapes: rectangle, square, circle, and octagon. Be sure to show your work and final answer labeled with the appropriate unit. Use additional paper as necessary. Final answers should be recorded on the table provided in question 10.

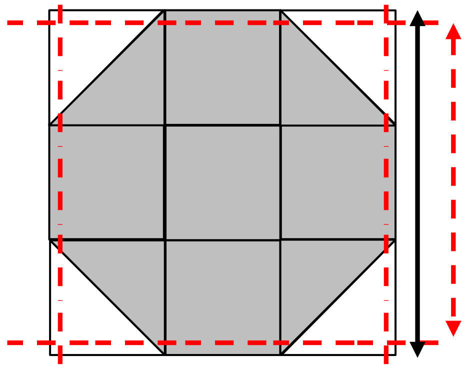
6. Find the area of a rectangular window with dimensions 4 ½ ft. by 3 ft. Include a labeled sketch of your window and show your work.

7. Find the area of a square window with dimensions 4 ft. by 4 ft. Include a labeled sketch of your window and show your work.

8. Find the area of a circular window with a diameter of 4 ft. Include a labeled sketch of your window and show your work.

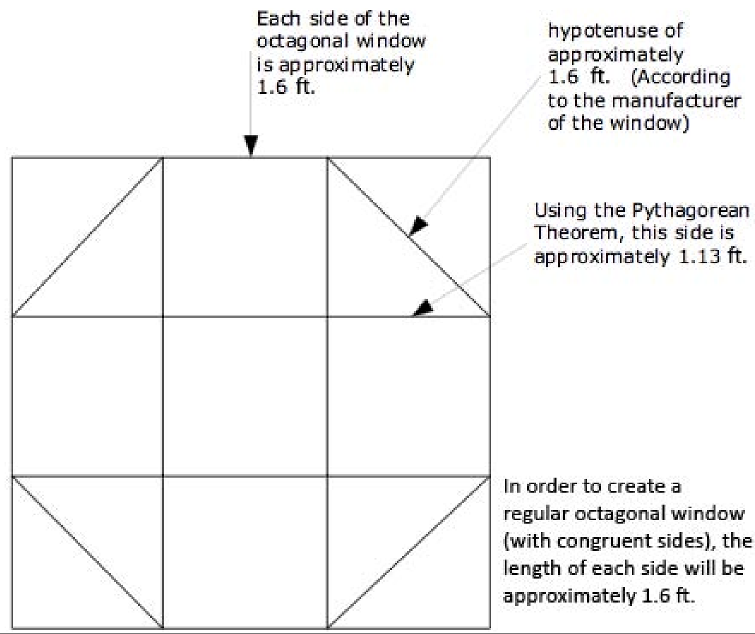
Octagonal windows are a bit more difficult to calculate. According to a window manufacturer, to fit an octagonal window into a square opening of 4 ft. by 4 ft., you would have to remove ¼ in. from all sides to accommodate the frame. This would leave you with an overall height (OH) of 3 ft. 11 ½ in. and an overall width (OW) of 3 ft. 11 ½ in.

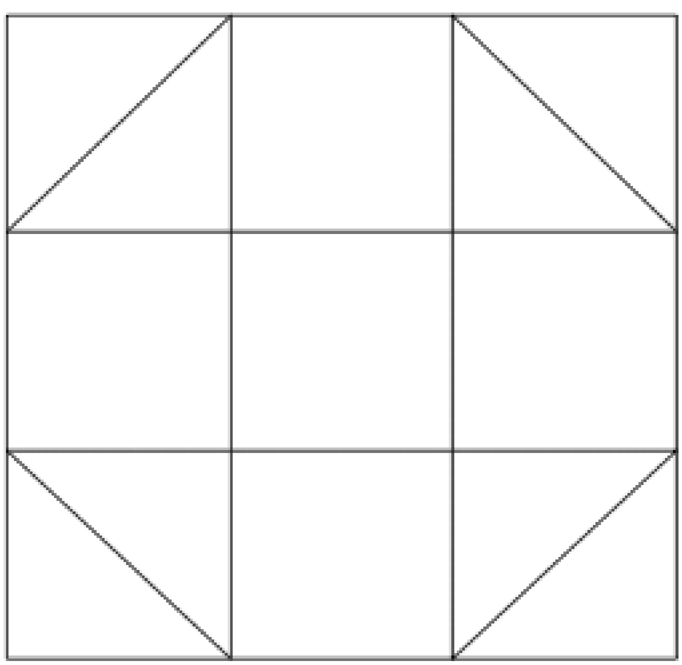
In order to more easily construct an octagonal window with all sides having the same length (congruent sides), this same window manufacturer stated that each side would have a length of approximately 1.6 ft. (approximately 1 ft., 7 in.).



Overall Height (OH) of the ***square*** *opening is 4 ft. (solid black arrow). If you subtract ¼ in. from each side to fit in an octagonal window the OH would be 4 ft. minus ¼ in. from the top and ¼ in. from the bottom which equals 3 ft. 11 ½ in. (red dashed arrow).*

The Overall Width (OW) would be computed the same way.





9. Let's think about this octagonal window. Looking at the sketch provided of the shaded octagon within the square, answer the following questions:

   A. What do you notice mathematically about this sketch?

   B. Describe how you would go about determining the area of the octagon.

   C. Find the area of this octagonal window that has an overall height (OH) of 3 ft. 11 ½ in. and an overall width of 3 ft. 11 ½ in. Include labels on the sketch of the window provided and show your work. Use additional paper as necessary.

10. Record the area of each shape you have determined. You will use this information later.

Window Shape with Dimensions	Area (in square feet)
4 ½ ft. by 3 ft. Rectangle	
4 ft. by 4 ft. Square	
Circle with 4 ft. Diameter	
Octagon with OH/OW of 3 ft. 11 ½ in.	

Oh, But the Cost!

Now that you have determined the area of windows with different dimensions and shapes, you will need to calculate the cost per square foot in order to help you make equal comparisons of the cost of your "perfect" window. Being cost effective may play a role in helping you determine the "perfect" window! Costs listed below are based on 2009 prices quoted by a local window distributor in Belgrade, Maine. Use the area for the windows that you previously computed, along with the listed total cost of the window to determine the cost (in dollars) per square foot. Show your computations in the work space provided. Use additional paper as necessary.

Image Credit:
http://doitbest.com/media/images/members/2244-window.jpg

Window Type (Shape & Size)	Area in square feet	Total Cost of the Window	Work Space	Cost (in dollars) per square foot
Rectangle (4 ½ ft. by 3 ft.)		$588.00		
Square (4 ft. by 4 ft.)		$465.00		
Circle (4 ft. diameter)		$1,662.00		
Octagon (side approximately 1.6 ft. or about 1 ft. 7 in.)		$648.00		

1. Use the information you have calculated in your table. Which window out of the four is the best buy? Explain how you determined this.

2. Would you ever choose a window that was not the best buy or one that was not the most cost effective? Explain.

Some people may consider purchasing a window that may not appear to be the best buy, if they are looking to add a more decorative flair. Stained glass windows, for example, can add a decorative element to any building. According to the Art Glass Association, the origins of stained glass probably came from jewelry making and mosaics. The art of stained glass became popular in the 10^{th} century, primarily in churches. Once made by adding metallic salts and oxides, this art glass is now uniquely crafted in a myriad of ways using new technologies. Due to the complexity of this process, stained glass windows may be quite expensive, smaller in size, and limited in shape selection. Let's do some comparative shopping with the costs you explored earlier and see if you might want your "perfect" window to include stained glass. (Remember to check your units!)

Image Credit: NASA Art Program

(Prices provided below are based on 2009 figures.)

3. Find the cost per square foot of a stained glass rectangle: 30 in. wide by 48 in. high at a total cost of $1, 306.95. Include a sketch and be sure to show your work.

4. What would the cost per square foot be of a stained glass square: 22 in. wide by 22 in. high at a total cost of $439.95? Include a sketch and be sure to show your work.

5. If the total cost was $414.95 for a stained glass circle with a diameter of 30 in., what would the cost be per square foot? Include a sketch and be sure to show your work.

6. Find the cost per square foot of a regular stained glass octagonal window with a side length of 18.75 in. with a total cost of $500.00. Include a sketch and be sure to show your work.

7. Now that you have had the opportunity to investigate windows, describe which of these would be your "perfect" window. Include the shape, dimensions, cost, and whether or not you are planning to use stained glass. Be sure to include a sketch of your window with labeled dimensions. Also describe the purpose of your "perfect" window.

MAKING AND MATHEMATICALLY EXPLORING YOUR OWN "STAINED-GLASS" WINDOW

You will now create your own "stained-glass" window using pattern blocks or tangrams to explore mathematical relationships of your stained-glass window design. Draw your window using colored pencils on a separate piece of paper. You must use at least 6 shapes (same or different shapes) for your window design. Once you have your own design created, you will mathematically explore your window.

1. Take one piece of your window, for example an equilateral triangle in the set of pattern blocks. Let this represent one base unit. Find the area of your window, using the piece you choose as one base unit. Be sure to identify your base unit below.

   One base unit = ___________________________________

   Sketch the window you created below, labeling the base unit piece. Find the area of your window. Be sure to show your work.

   Area of window = ___________________________________

2. Change your perspective by having another geometric piece be one base unit. Calculate the new area. Be sure to identify your new base unit.

   One base unit = ___________________________________

   Sketch the window you created below, labeling the base unit piece. Find the area of your window. Be sure to show your work.

   Area of window = ___________________________________

3. Describe your window mathematically. What shapes did you use? What percent of the total area does each shape represent of your stained glass window? What percent of the total area does each color represent of your stained glass window? What happened to the total area of your stained glass window as you changed the base unit? Include any other mathematical descriptions of your stained glass window. Remember to use proper mathematical vocabulary and good mechanics for your writing.

Oh, What a View!

Some windows take on a whole different perspective. Astronauts often view Earth from the "Destiny Module Science Window" when they are aboard the International Space Station. This window has the best optical quality ever placed on a human-occupied spacecraft. Astronauts take photographs, broadly referred to as "Earth Observations", that document human impacts on Earth such as city growth and agricultural expansion, natural events like hurricanes and floods, and surface features such as craters and volcanoes. Astronauts have been taking these pictures since the 1960's, forming an underlying foundation for the data collected by humans in space. Imagine how many pictures are in the NASA archives from all of their missions! Analyze the data collected from two time periods where data has been tabulated.

Data Tabulated	As of 1 April 2009	As of 26 May 2009
Number of missions in the catalog	164	166
Total images taken by astronauts	660,456	671,445
Total number of images taken from the International Space Station	354,852	365,169
Total number of images taken from the Space Shuttle	287,116	287,788

Answer each of the following questions using the data in the table above. Be sure to show your work.

1. What percentage of the total images taken through 1 April 2009 was taken from the Station?

2. What percentage of the total images taken through 26 May 2009 was taken from the Shuttle?

3. Look at the increased number of total images taken by astronaut from 1 April 2009 to 26 May 2009. What is the percent of increase?

Oh, What a Vision!

As you might have determined from the previous exercise, astronauts take a lot of images from space! They have actually used off-the-shelf film and digital cameras to take these images. So let's think about what we see with our eyes, compared to what a camera and lens enables us to see. Field of view or field of vision is the extent of what we can see at any given moment. Our eyes provide us with a field of vision. Once you put a camera up to your eye, your field of view changes. The camera lens now acts as our eyes. The focal length of the lens determines an area of coverage, or extent of what you can capture in an image. Astronauts select different camera lens sizes (shorter or longer lenses) based on how much area they wish to capture in the photograph.

This image is illustrating the area of coverage viewed when looking through different camera lenses.

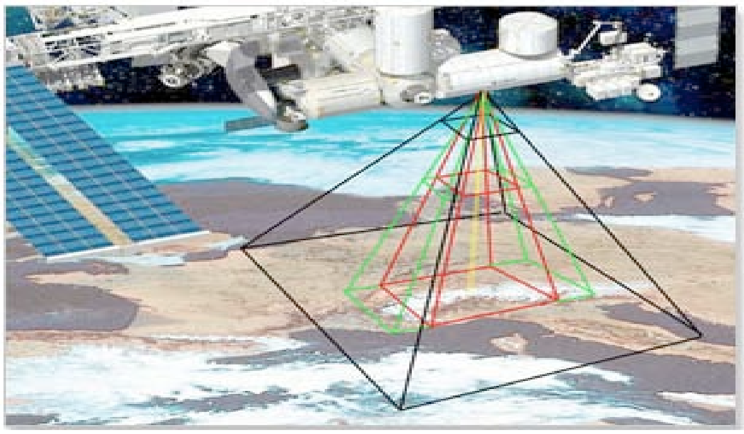


Image Credit: http://winearth.terc.edu/

Area of Coverage for a 70-300 mm Lens
70 mm = largest area of coverage
100 mm = large area of coverage
200 mm = small area of coverage
300 mm = smallest area of coverage

1. Based on the information above, what type of mathematical relationship explains the lens size and area of coverage?

2. The International Space Station's (ISS) inclination (or angle) of orbit was increased from 28.5 degrees to 51.6 degrees, significantly increasing the area of the Earth that would be visible to astronauts looking through the Destiny Window. What type of mathematical relationship does this describe?

3. Using the handout provided by your teacher, take a look at the sets of images taken with camera lenses of different focal lengths. Match the camera lens with the acquired astronaut photograph. Include a justification of your answer.

A. Image A1: Lens used: ______________________________________

Image A2: Lens used: ______________________________________

Write a brief justification of your answer:

B. Image B1: Lens used: ______________________________________

Image B2: Lens used: ______________________________________

Write a brief justification of your answer:

C. Image C1: Lens used: ______________________________________

Image C2: Lens used: ______________________________________

Write a brief justification of your answer:

D. Image D1: Lens used: ______________________________________

Image D2: Lens used: ______________________________________

Write a brief justification of your answer:

E. Image E1: Lens used: ______________________________________

Image E2: Lens used: ______________________________________

Write a brief justification of your answer:

F. Image F1: Lens used: ___________________________________

Image F2: Lens used: ___________________________________

Write a brief justification of your answer:

Mathematically describe either 2 individual images or a comparison between the two sets of images below. Be sure to include proportions or percentages of features, colors, or shapes visible in the image(s).

Mathematical Description #1
Image(s) being described: ____

Mathematical description:

Mathematical Description #2
Image(s) being described: ____

Mathematical description:

4. Suppose you are a scientist studying glaciers in South America. You first want an overall view of the glaciers in the region. What size camera lens would you request (give a range from the information provided) for your photographs and why?

5. After reviewing the images of this region, you notice a specific glacier of interest. You want to investigate the terminus (end) of this glacier in more detail. When you make your next request for photographs, what size lens would you request (give a range from the information provided) and why?

6. Consider how you might mathematically investigate a feature on Earth using astronaut photographs as you answer the following questions:
   A. If you had an opportunity to mathematically investigate a feature on Earth using astronaut photographs, what feature would you choose and why?

   B. Describe how you would mathematically investigate this feature.

   C. What benefit would a mathematical investigation be to you or someone else?